Optimal multicriteria approach to the iterative Fourier transform algorithm

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Abstract

In this paper, we propose a unified approach for the multicriteria design of diffractive optics. A multicriteria version of the Direct Binary Search (DBS) that allows the user to tune the compromise between diffraction efficiency and Signal to Noise Ratio already exists. This technique proves extremely powerful but also very time consuming. We extend this multicriteria approach to the Iterative Fourier Transform Algorithm (IFTA), which helps to reduce computation time dramatically, especially for multilevel domains. Simulations as well as experimental validations are provided.

**Keywords**: Diffractive Optical Elements, Computer Generated Holograms, multicriteria approach

**OCIS codes**: 090.1760, 090.1970

1. Introduction

Since the inception of diffractive optical elements (DOE’s) in the 1960’s, many powerful methods have been proposed to compute them within the framework of scalar diffraction theory. Nowadays, two kinds of techniques, both iterative, are mainly used: on the one hand, techniques
derived from the Direct Binary Search (DBS)\(^1\), and on the other hand, techniques derived from Projection Onto Convex Sets (POCS)\(^2\) or Gerchberg and Saxton’s algorithm\(^3\). Among the latter ones, the Iterative Fourier Algorithm Technique (IFTA) proposed by Wyrowski\(^4\) generally offers a good compromise between computation requirements and performance.

Legeard \textit{et al.}\(^5\) recently proposed an optimized multicriteria approach allowing the user to adjust a trade-off between diffraction efficiency \(\eta\) and amplitude error. This technique is based on a modified version of DBS that easily helps to deal with this trade-off adjustment. In this paper, we propose to extend this approach to IFTA in order to drastically reduce the computing requirements that DBS implies. We show that IFTA can be simply transformed into a tunable, easily tractable multicriteria technique, since the adjustment of a single parameter is required. DBS and IFTA can then be considered together in a single multicriteria approach.

\section{2. OT design basics}

The concept of optimal trade-offs was first presented to the optical community by Réfrégier\(^6\). Originally, it was applied to correlation filters and then extended by Legeard \textit{et al.}\(^5\) to diffractive optical elements. Optimal Trade-off (OT) DOE's\(^5\) are DOE's which, given N-1 criteria, optimize another criterion. The overall concept of OT's provides a practical framework for the study of DOE's because it allows any type of DOE to be included.
As suggested in the literature, let us consider an element $h$, not necessarily an optimal trade-off element in the beginning, taking account of the optical efficiency $\eta(h)$ and of the accuracy of the reconstruction (characterized by the amplitude error $Err_a(h)$) defined as:

\[
\eta(h) = \frac{A|g_1|^2}{M.N}
\]  

(1)

\[
Err_a(h) = \frac{\overline{A}[r_1 - \alpha|g_1|]}{A|r_1|^2}
\]  

(2)

with

\[
\alpha = \frac{A|g_1|}{A|g_1|^2}
\]  

(3)

where $f$ represents the desired pattern, $g$ the pattern reconstructed by the hologram $h$, $\mathcal{R}$ the Region Of Interest (ROI, the desired part of the reconstruction, not the whole reconstruction plane) and $M.N$ the number of pixels in the diffractive element. These criteria are clearly antagonistic.

A DOE $h_0$ is said to be an optimal trade-off DOE for the two criteria $\eta$ and $Err_a$, if there is no other DOE $h$ such as:

\[
\eta(h_0) \leq \eta(h) \text{ and } Err_a(h_0) < Err_a(h)
\]  

(4)
It is shown that such an element $\mathbf{h}$ minimizes the cost function

$$E_\mu(h) = \mu \cdot \frac{1}{\eta(h)} + (1 - \mu) \cdot Err_\mu(h)$$

(5)

where $\mu$ is a parameter characterizing the trade-off.

The concept of trade-off between several criteria provides an efficient way to compare the performance of DOE's and of their implementations onto various coding domains. The point is to draw the locus of points described by the different criteria when we explore all the possible implementations (with $\mu$ varying) onto a given coding domain $D$. This locus of points is usually convex (except in the case of a local minimum during the convergence) and is called the Optimal Characteristics Curve (OCC). Fig. 1 depicts a typical OCC shape when the considered criteria are efficiency and normalized error. For each OCC, the most interesting part of the curve depends on the constraints of the considered application, but it is often considered that the part close to the bottom right-hand corner is the most interesting: great improvement in diffraction efficiency can be obtained at the expense of a very slight degradation of error.

It should be noted that the OT general framework is not limited in the number of criteria and can be extended to global or spatial criteria such as reconstruction uniformity, noise reduction,… In this paper, for the sake of clarity, we will limit our study to the trade-off between the diffraction efficiency and the reconstruction accuracy which are the most major criteria in this context.
3. DOE design

3.1 Existing techniques

Two among the most powerful techniques for DOE's computation were incepted in the 1980's: Wyrowski and Bryngdahl proposed the Iterative Fourier Transform Algorithm (IFTA)\(^4,7,8\), whereas Seldowitz et al. proposed the Direct Binary Search (DBS)\(^1\).

IFTA, deriving from Gerchberg-Saxton algorithm\(^3\) and closely related to Projection Onto Convex Sets (POCS)\(^2\) and generalized projection onto convex sets (a generalized POCS onto sets which may not be convex)\(^9\), is known to provide a good compromise between the computation requirements and the accuracy of the reconstruction and so proves very popular. We shall describe it more precisely in the next Section.

DBS, a Monte-Carlo technique, is known to be computer intensive but also extremely accurate, usually more than POCS derived techniques\(^5,10\), especially when used with a simulated annealing optimization. Legeard et al.\(^5\) proposed to use IFTA as a preprocessing technique to DBS, which combines the accuracy of DBS with the reduced computer requirements of IFTA. Since DBS explicitly relies on a criterion optimization, it has been extended to a multicriteria version\(^5\) which consists in minimizing the cost function \(E_\mu(h)\) described in eq. (5).
3.2 Multicriteria IFTA

We have been interested in developing a multicriteria version of IFTA. Since IFTA, described Fig. 2 for the design of binary amplitude holograms, does not explicitly rely on a criterion optimization, it is not as straightforward as with DBS. In Wyrowski's papers\textsuperscript{7,8}, the goal is to provide an error-free reconstruction within the signal window, while maximizing the diffraction efficiency, possibly close to the theoretical maximum. Scale, amplitude and phase degrees of freedom are used to do so. Such techniques lead to the search of a unique solution. The multicriteria idea consists in loosening the constraint of producing an error-free reconstruction. Such an idea may sound strange, but provides interesting insight: the point is to introduce a supplementary degree of freedom, such that even the slightest loss of reconstruction accuracy can provide a high gain in diffraction efficiency. In that sense, we cannot claim that multicriteria IFTA is better than IFTA; it just provides extra solutions. For the same requirements, it exactly works in the same way, but it enlarges the IFTA range of applications when looser constraints than usual are applied.

Anyway, the first task is to find a parameter whose variations modify the convergence performance significantly, in terms of criteria. As described Fig. 2, IFTA consists in performing successive iterations between the spatial and spectral domains. The algorithm is divided into a given number of cycles of a given number of iterations. The spectrum constraint, i.e. the choice of the quantization bounds, is fixed over a cycle.
The possible parameters that we can tune are the number of cycles, the number of iterations per cycle and the scale factor $\beta_j$. \textit{A priori}, the number of iterations and the number of cycles do not play a significant role in the search of the compromise; a few simulations prove that they actually do not. On the contrary, $\beta_j$ which allows the noise to be rejected out of the ROI and so the error to be reduced, might play an important role in finding the compromise we are looking for. In previous works\textsuperscript{7}, $\beta_j$ is constrained to an optimized value, depending on the available degrees of freedom.

Actually, we want to use a parameter that makes sense in the OT framework, i.e. the parameter $\mu$. The idea is to apply a scale factor $\beta_j$ (depending on $\mu$) inside the ROI $\mathcal{R}$. If we state that $\mu=0$ corresponds to optimize only $Err_a$ and $\mu=1$ to optimize only $\eta$, then $\mu$ and $\beta_j$ should vary in the same direction.

Fetthauer \textit{et al.}\textsuperscript{11} had a similar idea, but they reported a mainly non-antagonistic behavior of the two criteria SNR and diffraction efficiency, which led them to consider, not a series of optimal trade-offs, but a unique compromise that seems optimal: their version of IFTA might be very different from ours, since IFTA should be seen rather as a framework, with many various derivatives and many parameters to tune, than as a definitive algorithm. Recently, Brenner\textsuperscript{12} proposed an updated version of Fetthauer’s idea consisting of a relaxed projection operator in the Fourier plane, known to provide stability during the convergence process. The relaxed projection parameter is optimized in order to vary over the cycles.
A few tries show rapidly that if \( \beta_j \) is lower than the unity, the algorithm diverges, leading to a null solution (it probably corresponds in ref.\(^{11} \) to the lowest values of the scaling factor \( \mu' \)) and is of no interest for us. We finally choose:

\[
\beta_j = (MN)^m
\]  

where \( MN \) represents the reconstruction space-bandwidth product. The power function in eq. (6) provides stability since \( \mu \) is positive-only and the term \( MN \) has been empirically determined in order to ensure scaling: a few tests have shown that no noticeable improvement in normalized criteria (larger than \( 10^{-3} \)) can be obtained with \( \beta_j \gg MN \), as detailed in Section 4.

Fig. 2 depicts the principle of IFTA and the way \( \beta_j \) is applied. We use a further parameter \( \beta_{\text{out}} \), multiplying the uninteresting part of the image (the part outside \( \Re \)) supposed to contain the noise rejected from the ROI by \( \beta_{\text{out}} \). The choice of an accurate value for \( \beta_{\text{out}} \) will be discussed in the next Section more precisely. Nevertheless, \( \beta_{\text{out}} \) will always be chosen lower than one, which implies the reduction of the relative importance of the noise (the part of the image outside the ROI) in comparison with the ROI. We use at the same time \( \beta_{\text{out}} \) outside the ROI and \( \beta_j \) inside the ROI. Therefore, the object constraint operator is expressed as:
\[
\mathbf{h}_k = \begin{cases} 
\mathbf{h}_k \cdot \mathbf{b}_j & \text{if } k \in \mathcal{S} \\
\mathbf{h}_k \cdot \mathbf{b}_{\text{out}} & \text{else}
\end{cases}
\]

(7)

Using simultaneously \( \beta_j \) and \( \beta_{\text{out}} \) as described before is not exactly equivalent to using the simpler modified operator:

\[
\mathbf{h}_k = \begin{cases} 
\mathbf{h}_k \cdot \mathbf{b}_j & \text{if } k \in \mathcal{S} \\
\mathbf{h}_k \cdot \mathbf{b}_{\text{out}} &= \mathbf{h}_k \cdot \frac{\mathbf{b}_{\text{out}}}{\mathbf{b}_j} & \text{else}
\end{cases}
\]

(8)

because of the nonlinearity of the quantization operator: the modified operator described in eq.(8), whereas producing a distribution similar to the one produced by the non-modified operator of eq.(7) (a scaled version, actually), modifies the overall energy of the reconstruction plane. As explained in Section 4, a few tries showed that the use of two independent parameters \( \beta_j \) and \( \beta_{\text{out}} \) instead of one \( \beta_{\text{out}} \) provided a faster convergence of the whole algorithm. The use of the operator described in eq.(7) will then be preferred.

4. Simulations

The diffractive elements presented in this paper are computed within the scalar diffraction theory framework and they require a thin convergent lens to be reconstructed: they are Fourier diffractive elements.
Since our reconstruction algorithm basically consists in applying a Fourier transform to the distribution describing our diffractive element, these elements may exhibit binary values (Section 4 and 5), multi-level values (Section 4) or continuous values (not addressed in simulations neither in experiments in this paper because at the present time, no available device allows the reproduction of continuous gray level values). No change in the reconstruction algorithm is required for this purpose.

All the simulations presented in this section concern 128*128 DOE’s which reconstruct various 32x32 test images. According to Legeard’s work\textsuperscript{13}, the choice of a value for $\beta_{\text{out}}$ depends on the constraints of the application. In this case, a binary Fourier diffractive element, the best choice is to choose $\beta_{\text{out}}$ equal to 0.85. Another choice, unless choosing $\beta_{\text{out}}$ greater than unity, will not make the algorithm diverge, it will only make it converge slower.

We first verified the point in varying $\mu$ to tune the compromise between optical efficiency and error. As expected, if $\mu$ is chosen greater than one, the algorithm diverges rapidly. The value of error obtained for $\mu=0$ is the lowest attainable with IFTA. For values of $\mu$ between 0 and 1, efficiency increases, at the expense of an increasing error. For negative values of $\mu$, the behavior obtained is similar to the one reported by Fetthauer et al. (since it is not an optimal trade-off behavior, we do not report it in the following). The evolution of diffraction efficiency vs. $\beta_j$ is reported Fig. 3; it clearly shows the links between both parameters.
Fig. 4 depicts the loci of points ($\eta$, error), obtained for various techniques and binary DOE’s, and Table 1 reports the corresponding computation times for 128*128 DOE’s, whether they are binary amplitude or 4 phase level elements. Since the locus of points ($\eta$, error) is now a full curve instead of a single point (as for the classic IFTA), it demonstrates that we have indeed obtained an optimal trade-off behavior. Moreover, it clearly shows that, when an iterative equalizer is used, the performance obtained with IFTA is close to that provided by DBS, or even better, whereas the computation time has been considerably reduced. Except for extreme cases, when a very low error or a high efficiency is required, this makes IFTA the best possible candidate for DOE computation. As suggested by Legeard et al.\textsuperscript{5}, the equalizer used in this paper corresponds to an IFTA diffuser\textsuperscript{14} with a modified spectrum constraint consisting of variable clipping. Hybrid input-output iterations\textsuperscript{15} were also introduced for better and faster convergence. They consist of a Gerchberg-Saxton algorithm with a relaxed projection in the reconstruction plane as:

$$g_{i+1}(k) = \begin{cases} (1 - \gamma)g_i(k) + g_f(k) & \text{k iterations} \\ \hat{g}_i(k) & \text{else} \end{cases}$$

where $\gamma$ is the relaxation parameter, $k$ a pixel location and $i$ describes the iteration rank.

Computation times reported for the 4 phase level elements show the huge interest of multicriteria IFTA over multicriteria DBS: in the case of IFTA, these computation times are almost completely independent of the number of levels $N_l$, whereas they grow like $N_l$ with DBS.

Another step in the optimal trade-off design would have been to evaluate the criteria for a given parameter, to accept or reject the iterations according to the corresponding variations and then to
resume the algorithm with a modified parameter. We tested that algorithm: it did not improve the results significantly since IFTA naturally and regularly converges to the desired minimum, so we came back to the original multicriteria IFTA described above.

5. Experimental validation

We tested DOE's computed with multicriteria IFTA on a twisted nematic liquid crystal display from CRL referenced SVGA1. Though such devices show gray level modulation ability\(^\text{16}\), we wanted to demonstrate that our technique works in the most stringent case, i.e. the binary case. 256x256 elements, reconstructing the same gray level 153x60 pattern, were computed and tiled to the modulator space bandwidth product, 800x600, in order to reduce speckle. The SVGA1 modulator shows a 33 µm pixel pitch with a 0.57 fill factor and is operated at 60 Hz with a 632.8 nm He-Ne radiation.

Fig. 5 gathers several views of the reconstruction plane for various values of \(\mu\), showing slightly degrading reconstructions when \(\mu\) increases. However, these impressions are not sufficient to prove the optimal trade-off behavior of our technique and we performed an objective evaluation of its performance through precise measurements.

The summed point to point error \(Err\) was not evaluated, since it is known to give poor experimental figures, far from the real quality of the reconstructions. Incidentally, the question of evaluating the experimental reconstructions of diffractive elements has not been solved yet in a convincing manner. To remove speckle, the elements are usually replicated, which leads to a
sampled reconstruction plane. If we want to retrieve $Err_a$ figures from this reconstruction plane, we need a camera and the reconstruction sampling period must be an exact multiple of the camera sampling period, in order to compute the sum of local differences expressed in eq. (2). Thus, the comparison of simulated and experimental figures does not prove straightforward, and this is probably why this complicated technique has never been reported, to our knowledge.

Instead, we considered the Noise Reduction Ratio (NRR), as suggested by Moreno et al., expressed as follows:

$$NRR = \frac{E_{W(n,o)}}{E_{W(s,i)}}$$  \hspace{1cm} (10)

where $W(n,o)$ depicts a window containing noise outside the reconstruction window, $W(s,i)$ depicts the reconstruction window, i.e. the region of interest $\mathcal{R}$, and $E_{W(.,.)}$ the energy evaluated within these windows. The NRR criterion should be used carefully. The less noise (and energy) outside the reconstruction window, the more energy within the reconstruction window. Then, the diffraction efficiency increases when $E_{W(n,o)}$ decreases, but nothing ensures that this additional energy does not degrade the desired reconstruction.

We then evaluated a criterion that we named the Simplified Signal to Noise Ratio (SSNR). This criterion corresponds to an evaluation of the error, summed over a window $W(n,i)$ inside the reconstruction window $W(s,i)$. To easily evaluate the error within $W(n,i)$, we chose the latter one as a zone where the reconstructed signal should be zero. Then, SSNR is expressed as:
All the windows used in our experimental measurements are shown Fig. 6. We measured \( E_{W(x,i)} \), \( E_{W(n,i)} \) and \( E_{W(n,o)} \) for various values of \( \mu \). Fig. 7 reports the loci of points \((\eta, 1/NRR)\) and \((\eta, 1/SSNR)\) for experimental data and Fig. 8 the locus of points \((\eta, \text{error})\) for simulated data. In both cases, the diffraction efficiency is not scaled as eq.(1) suggests it, but as it can be seen as an energy ratio, and provided that the incident light does not vary, it has been replaced by the experimental measurement of the gray level mean value of the energy in \( \mathbb{R} \). Both figures clearly show an optimal trade-off behavior. We first notice that, as anticipated, NRR sometimes fails in evaluating the accuracy of the reconstruction correctly, because it considers all the energy in the reconstruction window in the same manner, whether it is noise or not. Instead, this figure shows the antagonistic behavior of SSNR and efficiency. It clearly appears that when \( \mu \) increases, we obtain an important improvement in efficiency at the expense of a slight degradation of SSNR, often hardly noticeable in the optical reconstructions: this is a typical optimal trade-off behavior. Further simulations and experiments show that this behavior is marked more strongly (the OCC looks more like the typical shape of Fig. 1) when an iterative IFTA-like diffuser is used; but in that case, as in Fig. 4 with our test images, the variation range in efficiency is smaller.

Table 2 allows the comparison of the simulated and experimental implementations. For a given figure, we define the range ratio as the ratio of the maximum and minimum value of the figure. Efficiency and error range ratios have comparable respective numerical values in simulation and in experimentation.

\[
SSNR = \frac{E_{W(x,i)}}{E_{W(n,i)}}
\]
6. Conclusion

In this paper, we propose a unified approach for the multicriteria design of diffractive optics. The Direct Binary Search technique has already proved its flexibility and its ability to be easily transformed into a multicriteria method, allowing the user to find a trade-off between diffraction efficiency and reconstruction accuracy. We extend this classic multicriteria approach to IFTA, which helps to dramatically reduce the computation times, especially for multilevel domains, without sacrificing the reconstruction accuracy or the diffraction efficiency. So, without increasing computation time, we turned IFTA into an optimal trade-off technique whose performance can adapt to the user's requirements and can equal the performance provided by DBS, whereas this latter proves extremely computer intensive. The experimental reconstructions obtained when binary optimal trade-off DOE's are displayed on a twisted nematic liquid crystal SLM confirm the simulations through the evaluation of efficiency and of a simplified noise criterion.

Acknowledgments

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Fig. 1: a typical Optimal Characteristics Curve (OCC) for the criteria (\(\eta, Err_\eta\)) for \(\mu\) varying from 0 to 1. The most interesting part is highlighted.

Fig. 2: principle of IFTA for the design of binary amplitude holograms. \(\beta_{\text{out}}\) is applied in the object plane, in \(\mathbb{R}\) (i.e. outside \(\mathbb{R}\)), and is smaller than one. \(\beta_j\) applies to \(\mathbb{R}\) only. In classic IFTA, \(\beta_j\) is fixed to one, whereas it varies in multicriteria IFTA, as explained in Section 3. ‘FT’ stands for ‘Fourier transformation’.

Fig. 3: diffraction efficiency vs. \(\beta_j\) in the case of a binary amplitude coding-domain.

Fig. 4: loci of points (\(\eta,\) error) for \(\mu\) varying from 0 to 1, for various techniques and for binary DOE’s. For both multicriteria IFTA curves (i.e. with and without equalizer), the point corresponding to the non-multicriteria case is highlighted in a bigger pattern.

Fig. 5: various reconstructions produced by SVGA1 SLM operated in binary mode. (a), (c), (e) and (g) report simulations with the respective parameters \(\mu\) of 0, 0.01, 0.05 and 0.999, and (b), (d), (f) and (h) corresponding to optical reconstructions. When \(\mu\) increases, a degradation in image accuracy as well as an increase in diffraction efficiency can be noticed.
Fig. 6: three windows used for our criteria evaluation. $W(s;i)$ is the signal window (region of interest), $W(n,i)$, used for SSNR, is a noise window inside the signal window and $W(n,o)$, used for NRR, is a noise window outside the signal window.

Fig. 7: locus of points ($\eta$, $\text{NRR}$) and ($\eta$, $\text{SSNR}$) for $\mu$ varying from 0 to 1, corresponding to the experimental reconstructions presented Fig. 5. The experimental efficiency $\eta$ is defined as the mean energy, expressed in terms of gray level value, in the region of interest $\mathcal{R}$. As highlighted, using $\text{NRR}$ may result in an incorrect evaluation of the reconstruction accuracy.

Fig. 8: Locus of points ($\eta$, SNR) for the simulated reconstructions presented Fig. 5.
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Fig. 5 (subfigures (a) to (d))
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<th></th>
<th>Binary DOE’s</th>
<th>4-level phase DOE’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal.+IFTA+DBS</td>
<td>160-630 s</td>
<td>500-1,300 s</td>
</tr>
<tr>
<td>DBS</td>
<td>150-570 s</td>
<td>430-2,000 s</td>
</tr>
<tr>
<td>Multicriteria IFTA</td>
<td>25 s</td>
<td>25 s</td>
</tr>
<tr>
<td>Equalizer + Multicriteria IFTA</td>
<td>75 s</td>
<td>75 s</td>
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</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th>simulated data</th>
<th>experimental data (1/SSNR)</th>
<th>experimental data (1/NRR)</th>
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<tbody>
<tr>
<td>efficiency range ratio</td>
<td>1.42</td>
<td>1.27</td>
<td>1.27</td>
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<tr>
<td>error range ratio</td>
<td>2.66</td>
<td>2.22</td>
<td>2.17</td>
</tr>
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